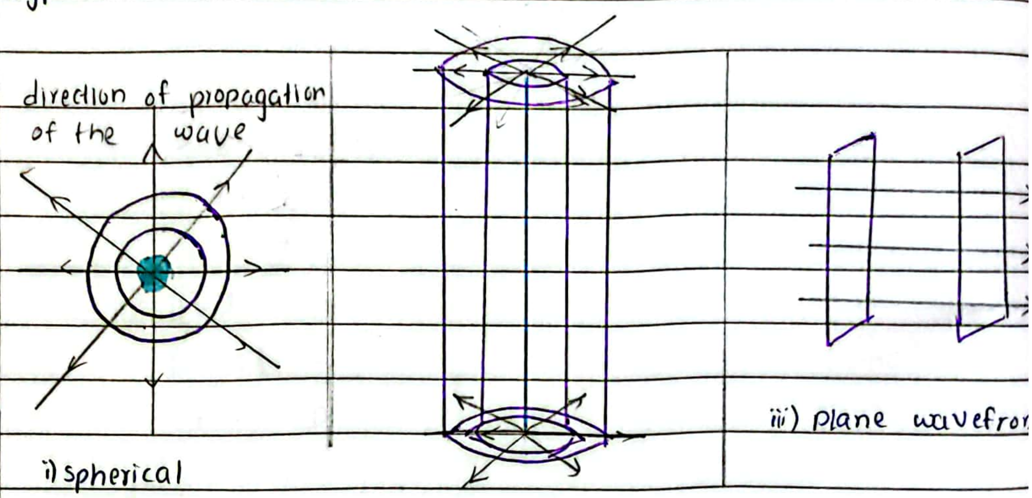
# CH: WAVE OPTICS

Wave optics is a branch of physics which deals with the stu
of light on the basis of the wave nature of light

**Wavefront:**

- The locus of all points which are vibrating in the same phase i
called a wavefront.
- A wavefront is always ⊥ to the ray of light.

Types of wavefront




i) spherical wavefront

ii) Cylindrical wavefront

iii) Plane wavefro

Direct anl.

## Huygen's Principles

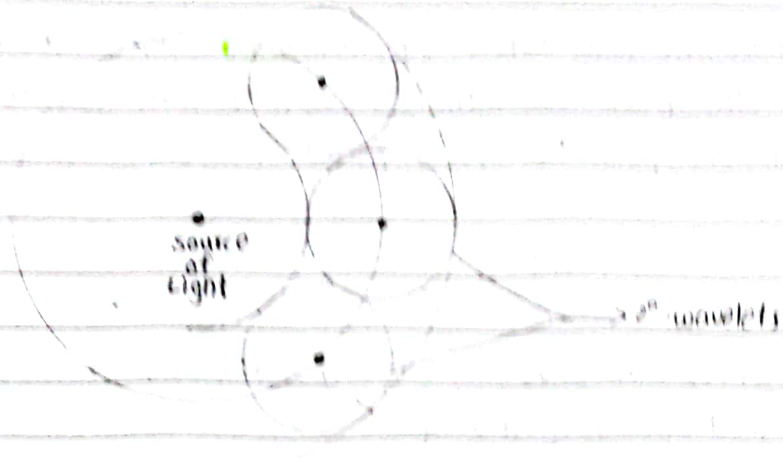
1st Principle

Every point on a wavefront acts as a fresh source of light which emits light in all directions.

- The wavefronts of such fresh sources of light are k/a secondary wavelets.

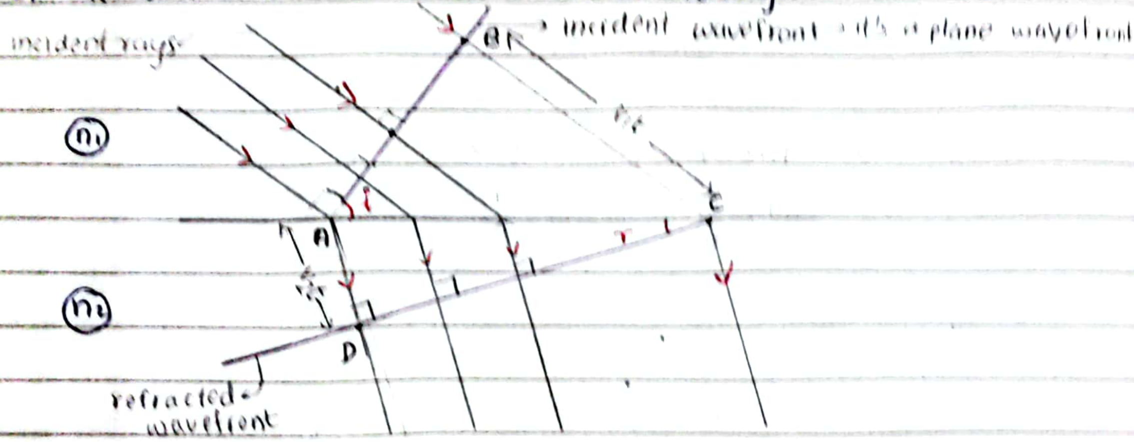
2nd Principle

The surface tangential to the secondary wavelets at any inst gives the position of the new wavefront at that instant.




Board 5M.

Laws of Refraction on the basis of wave theory




In figure, AB is the incident wavefront in a medium of refractive index $n_1$ and CD is the refracted wavefront in a medium of refractive index $n_2$. $i$ is the angle of incidence and $r$ is the angle of refraction.

From $\triangle ABC$,

$$\sin i = \frac{BC}{AC} \longrightarrow (1)$$

From $\triangle ADC$

$$\sin r = \frac{AD}{AC} \longrightarrow (2)$$

$$\frac{(1)}{(2)} \quad \frac{\sin i}{\sin r} = \frac{BC}{AD} \longrightarrow (3)$$

let t be the time taken by the wave to reach the point from the point B, then the time taken by the wave reach the point D from point A, will also be t

$$\therefore BC = v_1 t$$

$$AD = v_2 t$$

where $v_1$ & $v_2$ are the speeds of light in 1st & 2nd medium respectively

$\therefore$ ③ becomes $\Rightarrow \frac{\sin i}{\sin r} = \frac{v_1 t}{v_2 t} \longrightarrow$ ④

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} \longrightarrow ⑤$$

we have the refractive index

$$n = \frac{c}{v}$$

$$v = \frac{c}{n}$$

$$v_1 = \frac{c}{n_1}, \quad v_2 = \frac{c}{n_2}$$

$\therefore$ eqn ⑤ $\frac{\sin i}{\sin r} = \frac{c/n_1}{c/n_2}$

$$\boxed{\frac{\sin i}{\sin r} = \frac{n_2}{n_1}} \longrightarrow ⑥$$

This is Snell's law of Refraction.

NOTE: Let $t = T$ ($T \Rightarrow$ time period of wave)

$\therefore$ ④ $\Rightarrow \frac{\sin i}{\sin r} = \frac{\overset{\text{velocity}}{v_1 T}}{v_2 T}$

$$\frac{\sin i}{\sin r} = \frac{\lambda_1}{\lambda_2} \longrightarrow ⑦$$

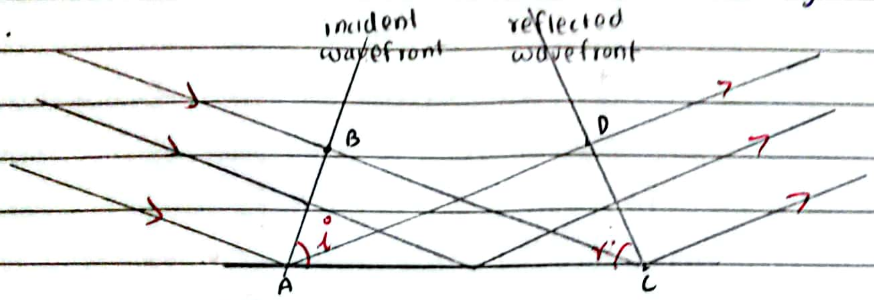
From ⑥ & ⑦

we get $\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2}$ • inversely proportional

$$\boxed{n_1 \lambda_1 = n_2 \lambda_2}$$

$\therefore n\lambda \rightarrow$ a constant

Law of Reflection on the basis of wave theory:-




In the figure AB is the incident wavefront and CD is the reflected wavefront. $i$ is the angle of incidence and $r$ is the angle of reflection.

From $\triangle ABC$ and $\triangle ADC$

$\angle BAC = \angle DCA$

$\therefore i = r$ //

# Interference of Light

Coherent sources of light

2 sources of light are said to be coherent if they emit lights of same frequency and with a zero phase difference or a constant phase difference.

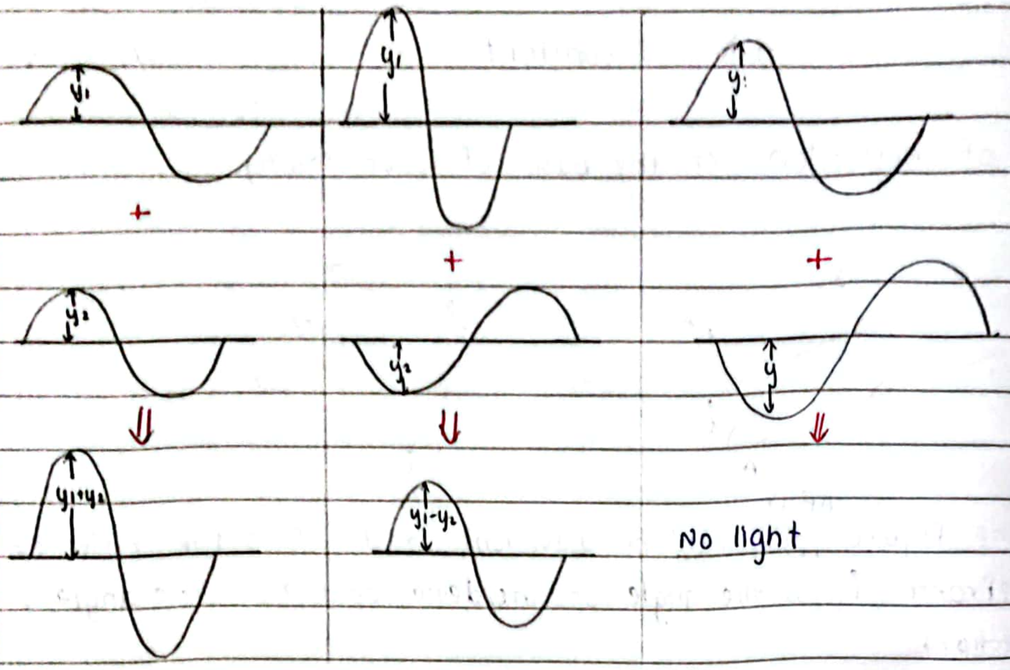
Path Difference & Phase Difference

| Path diff. | Phase diff |
|---|---|
| $\lambda$ | $2\pi$ |
| 1 | $2\pi/\lambda$ |
| $x$ | $2\pi x/\lambda$ |
| $\lambda/2$ | $\pi$ |

## Superposition Principle

When 2 or more waves are superimposing each other, the resultant displacement is equal to the algebraic sum of displacements produced by individual waves i.e

$$y = y_1 + y_2 + y_3 \cdots\cdots$$

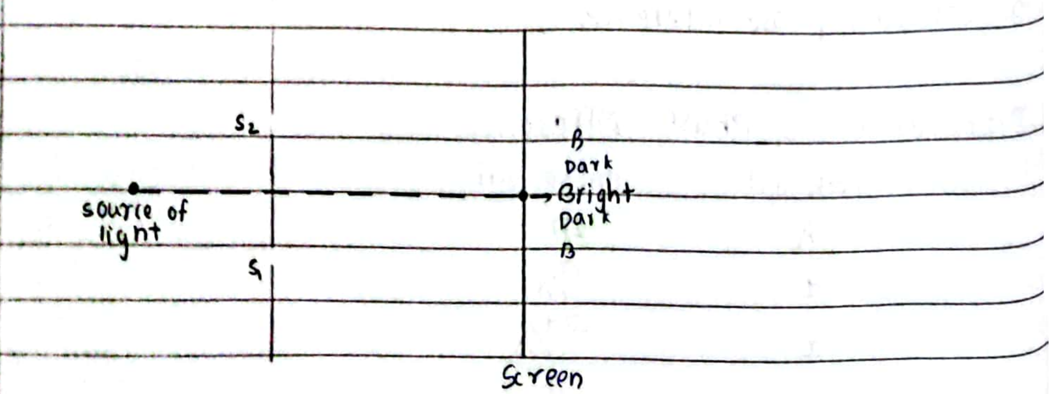



## Interference of Light

The phenomenon of redistribution of light energy due to the superposition of 2 or more waves is called the interference.

Young's Double slit experiment (YDSE)

The experimental set up consists of 2 narrow slits $S_1$ & $S_2$, kept in between a source of light and a screen.



On the screen, one can observe alternate bright & dark fringes with a bright fringe at the centre

The pattern is called interference pattern.

Conditions for Bright & Dark fringes

Consider 2 waves $y_1 = A\sin\omega t$ and
$y_2 = A\sin(\omega t + \phi)$,
superimposed to produce an interference pattern on a screen where $\phi$ is the phase difference b/w the 2 waves meeting at a point.

Let $I_0$ be the intensity of the incident wave,
Then $I_0 \propto A^2$

The intensity of the resultant wave is given by,

$$\boxed{I = 4 I_0 \cos^2\left(\frac{\phi}{2}\right)} \rightarrow \text{phase difference}$$

I For Bright Fringe → band

$\cos^2 \phi/2 = 1$

$\cos \phi/2 = \pm 1$

$\phi/2 = 0, \pi, 2\pi, 3\pi \ldots$

$\phi = 0, 2\pi, 4\pi \ldots$ $\quad \because \lambda = 2\pi$

$\phi = n(2\pi)$

$\phi = n(\lambda)$

The condition for bright fringe is

$$\boxed{\text{Path difference} = n\lambda}$$

$n = 0, 1, 2, 3 \ldots$

For Dark Fringe:

$$\cos^2 \phi/2 = 0$$

$$\cos \phi/2 = 0$$

$$\phi/2 = \pi/2, 3\pi/2, 5\pi/2$$

$$\phi = \pi, 3\pi, 5\pi$$

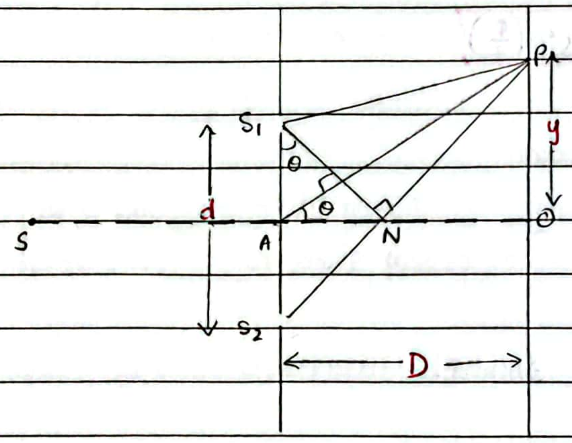
$$\phi = (2n-1)\pi$$

The condition for dark fringe is

$$\boxed{\text{Path difference} = (2n-1)\frac{\lambda}{2}}$$

$n = 1, 2, 3 \ldots$

## EXPRESSION FOR FRINGE WIDTH




DON'T STOP

Consider a YDSE setup consisting of 2 slits $S_1$ and $S_2$ seperated by a small distance 'd'. The screen is placed at a distance 'D' from the slits. P is a point on the screen at a distance 'y' from the central bright fringe

From $\Delta S_1 S_2 N$

$$\sin\theta = \frac{S_2 N}{S_1 S_2}$$

$$\theta = \frac{S_2N}{d} \rightarrow (1)$$

From $\triangle AOP$

$$\tan\theta = \frac{OP}{AO}$$

$$\theta = \frac{y}{D} \rightarrow (2)$$

From (1) & (2)

$$\frac{S_2N}{d} = \frac{y}{D}$$

$$y = \frac{D}{d} \times S_2N$$

$$\boxed{y = \frac{D}{d} \times \text{Path difference}}$$

(1) For bright:

Path diff. $= n\lambda$

$$\therefore y = \frac{D}{d} \times n\lambda$$

Let $n = 1$

$$y_1 = \frac{D\lambda}{d}$$

Let $n = 2$

$$y_2 = \frac{2D\lambda}{d}$$

$\therefore$ Width of dark fringe : $\beta = y_2 - y_1$

$$= \frac{2\lambda D}{d} - \frac{\lambda D}{d}$$

$$\boxed{\beta = \frac{\lambda D}{d}}$$

II For Dark fringe

Path diff $= (2n-1)\frac{\lambda}{2}$

$$\therefore y = \frac{D}{d} \times (2n-1)\frac{\lambda}{2}$$

Let $n=1$

$$y_1 = \frac{D}{d} \times \frac{\lambda}{2}$$

Let $n=2$

$$y_2 = \frac{3}{2}\frac{D\lambda}{d}$$

$\therefore$ Width of bright fringe : $\beta = y_2 - y_1$

$$\beta = \frac{3}{2}\frac{D\lambda}{d} - \frac{1}{2}\frac{D\lambda}{d}$$

$$\beta = \frac{1}{2}\frac{D\lambda}{d}(3-1)$$

$$\boxed{\beta = \frac{D\lambda}{d}}$$

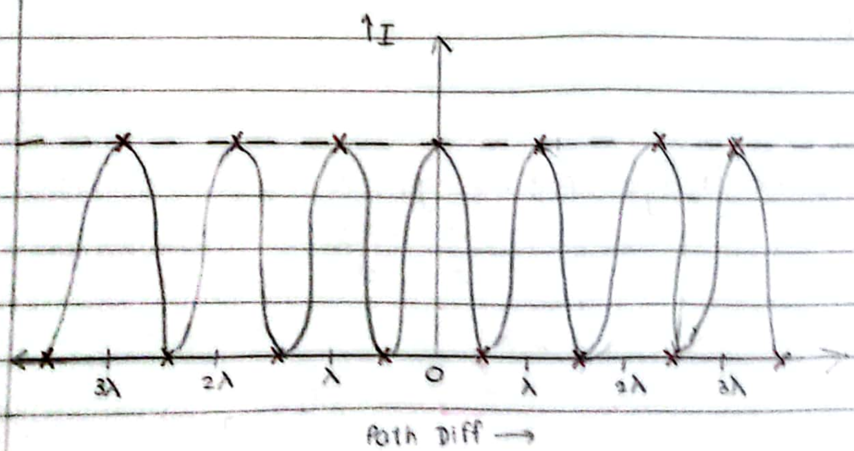
The fringe width in interference pattern depends on :

i) wavelength of light

ii) distance between the slits and the screen.

iii) distance between the two slits.

INTENSITY DISTRIBUTION CURVE

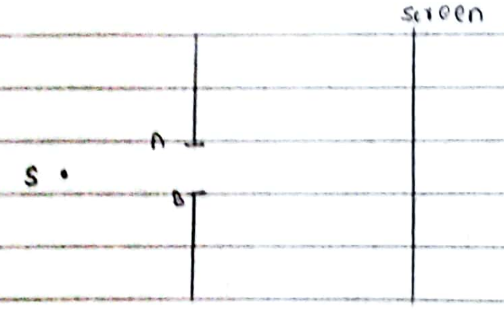


# DIFFRACTION

The phenomena of bending of light around the edges or corners of an obstacle is called the diffraction of light.

## Diffraction at a single slit :-

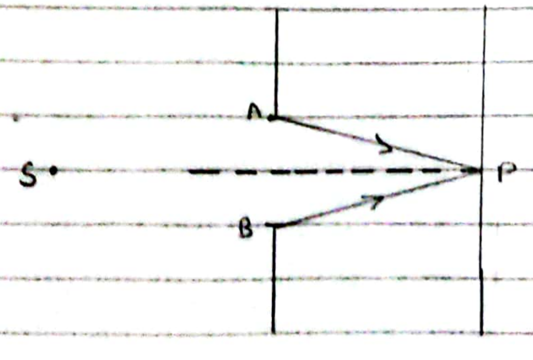
Consider a narrow slit AB kept in between a source of light and a screen




On the screen, One can observe alternate bright and dark rings with a bright spot at the centre.
The pattern obtained is called the diffraction pattern.

- The central bright spot with maximum intensity is known as the central maxima.
- The other bright rings with less intensities are called secondary maximas. and the dark rings are called secondary minimas.

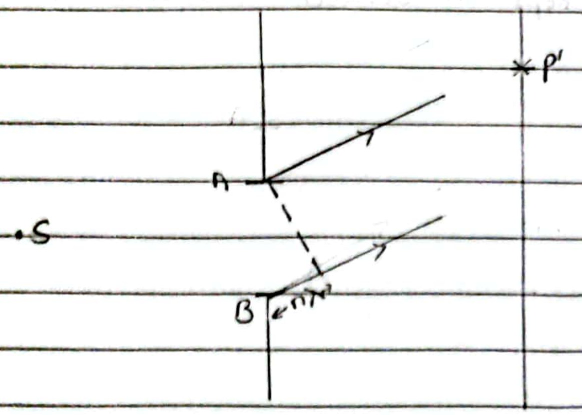
## CENTRAL MAXIMA



Let P be the midpoint of the screen, then the path difference between 2 waves reaching the point P from A and B will be 0.

Therefore, these 2 waves will interfere constructively to produce a bright spot at the centre called the central maxima.

## SECONDARY MINIMAS:-



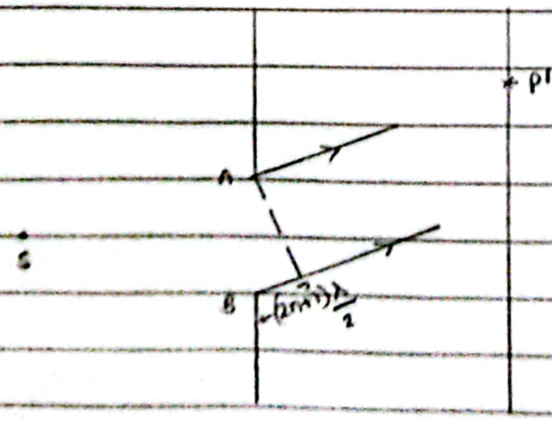

Consider any point P' on the screen. The point P' will be dark if the path difference between AP' and BP' is equal to $n\lambda$

$\therefore$ The condition for secondary minima is

$$\boxed{\text{Path difference} = n\lambda}$$

$n = 1, 2, 3, \dots$

## SECONDARY MAXIMAS



The point P' will be bright if the path difference between AP' and BP' is equal to $(2n+1)\frac{\lambda}{2}$

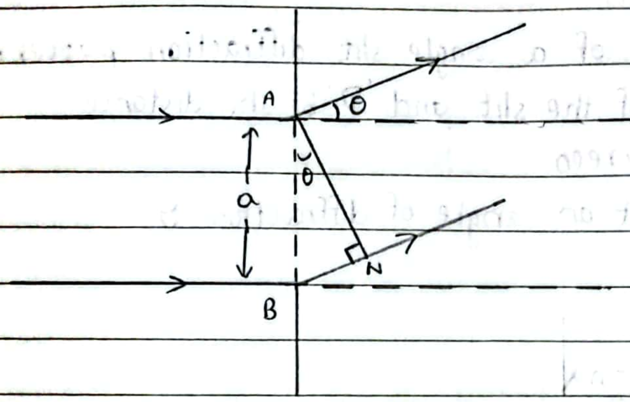
∴ The condition for secondary maxima is

$$\boxed{\text{Path difference} = (2n+1)\lambda/2}$$

$n = 1, 2, 3 \ldots$

## ANGLE OF DIFFRACTION (θ)

Consider a small slit of width a.




Let θ be the angle of diffraction

From $\triangle ABN$,

$$\sin\theta = \frac{BN}{AB}$$

$$BN = AB\sin\theta$$

$$\boxed{\text{Path difference} = a\sin\theta}$$

NOTE: For minima

$$\text{Path difference} = n\lambda$$

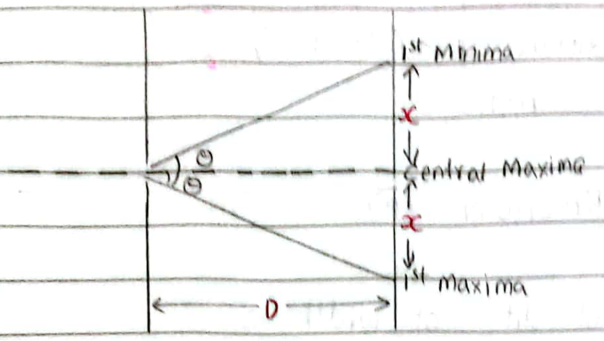
$$a\sin\theta = n\lambda$$

$$\boxed{\sin\theta = \frac{n\lambda}{a}}$$

$$\therefore \theta \propto \frac{\lambda}{a}$$

i.e to observe the diffraction pattern, the size of the obstacle must be comparable with wavelength of light.

## EXPRESSION FOR THE WIDTH OF CENTRAL MAXIMA:-




Consider the formation of a single slit diffraction pattern. Let 'a' be the width of the slit and 'D' be the distance between the slit and screen.

1st minima is formed at an angle of diffraction θ

For minima,

Path difference $= n\lambda$

$a\sin\theta = n\lambda$ ($\because n=1$ for 1st minima)

$a\sin\theta = \lambda$

$a = \frac{\lambda}{\sin\theta}$

$\sin\theta = \frac{\lambda}{a}$ ($\because$ θ is very small)

$\theta = \frac{\lambda}{a} \rightarrow ①$

From fig : $\tan\theta = \frac{x}{D}$ ($\because$ θ is very small)

$\theta = \frac{x}{D} \longrightarrow ②$

From ① & ②

$\frac{\lambda}{a} = \frac{x}{D}$

$$x = \frac{D\lambda}{a}$$

- Width of central maxima

$$W = 2x$$

$$\boxed{W = \frac{2D\lambda}{a}}$$

F: Angular width $= 2\theta$

$$= \frac{2\lambda}{a}$$

NOTE: The angular width is independent of the distance between the slit and the screen

INTENSITY DISTRIBUTION CURVE